Asset pricing with beliefs-dependent risk aversion and learning

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戴外林

山西财经大学

Tony Berrada

His research interests are in asset pricing, and in particular the role of learning in models with incomplete information.

- He is a Professor of Finance at the University of Geneva (Geneva Finance Research Institute and Swiss Finance Institute).
- He hold a PhD in finance from the University of Lausanne and was an assistant professor of finance at HEC Montréal and HEC Lausanne prior to joining the University of Geneva.
- He also was a senior visiting fellow at the School of Banking and Finance, Australian School of Business at UNSW

Jérôme Detemple

Education
Wharton School, University of Pennsylvania: PhD, Finance, December 1983; MA, 1982
UniversitÈ Paris-Dauphine: DEA. Finance, Magna Cum Laude, 1980
ESSEC(法国著名学府埃塞克高等商学院):MA, 1979
Publication
On American VIX(波动率指数) Options under the Generalized 3/2 and1/2 Models
American Options with Two-level Caps
Optimal Investment under Cost Uncertainty

Marcel Rindisbacher

Education

1994 - 2000 Ph.D. in Economics, ″ Trois essais en ´economie financier″ , Universit´e deMontr´eal.

1992 - 1994 M.Sc. in Econometrics and Mathematical Economics, LSE(伦敦政治经济学院) UK

1986 - 1992 B.A. in Economics (lic.rer.pol.), Universit¨at Bern, Switzerland

Publications

Asset Pricing with Regime Dependent Preferences and Learning

A Structural Model of Dynamic Market Timing

Lifecycle ConsumptionInvestment Policies and Pension Plans: A Dynamic Analysis

explanation of title

- Asset pricing
- The market price of risk, the interest rate, and the stock return volatility
- beliefs-dependent risk aversion
- learning
- Innovation in information

abstract

- This paper studies equilibrium in a pure exchange economy with unobservable Markov switching growth regimes and beliefs-dependent risk aversion (BDRA).
- The market price of risk, the interest rate, and the stock return volatility acquire new components tied to fluctuations in beliefs.
- A three-regime specification is estimated using the generalized method of moments (GMM).
- A new factor, the information risk premium, is found to be a strong predictor of future excess returns.

Introduction

- puzzles of the classic intertemporal asset pricing model.
- 1、The excess volatility phenomenon.
- 2、The equity premium puzzle.
- 3、The abnormally high level of the interest rate.

This paper seeks to address these fundamental issues

- The model developed has three key ingredients
- unobservable growth regimes.
- beliefs-dependent risk aversion.
- information structure:information from consumption, dividends, and macroeconomic variables, such as unemployment.

unobservable growth regimes

- the underlying growth regime evolves according to a Markov switching process with a finite number of states ($s_1, . . ., s_K$), but is not observed.
- Nonobservability Uncertainty p
- Information Innovations in information（learning）$p_t = (p_{1t}, . . ., p_{Kt})$（条件概率）
- This learning mechanism constitutes an additional source of variability on top of the usual consumption channel.

beliefs-dependent risk aversion

- Beliefs-dependent utility,broadly defined, describes a preference model in which the utility of consumption depends on the probability of events.
- $p_t = (p_{1t}, \ldots, p_{Kt})$（后验信念） RA

$$u(c_t, p_t, t) = \sum_{k=1}^{K} \frac{1}{1-R_k} e^{-\beta t} c_t^{1-R_k} p_{kt}$$

information structure

- information is derived from consumption, dividends, and macroeconomic aggregates.
- the data uses unemployment（macroeconomic）
- Macroeconomic information（unemployment ） is the source of a novel premium component, called the information risk premium (IRP).

market price of risk

- market price of risk has now three components corresponding to consumption risk, dividend risk, and macro risk.
- The last two components are new and directly tied to the dependence on beliefs. They vanish in the absence of an interaction between consumption and beliefs
- The interest rate inherits three new components, all tied to the effect of beliefs on instantaneous utility.
- As for the volatility of the market return, it has one additional component, reflecting the impact of macro information on the volatility of beliefs.

Relative risk aversion

- Relative risk aversion, in the model examined, is

$$R_t = -\frac{u_{cc} c_t}{u_c} = \sum_{k=1}^{K} R_k q_{kt},$$

where

$$q_{kt} = \frac{c_t^{-R_k} p_{kt}}{\sum_{k=1}^{K} c_t^{-R_k} p_{kt}}.$$

The economy

- Uncertainty is described by (W, s), where W ≡ (W^C, W^G,W^Y) is a trivariate standard Brownian motion process and s is a continuous time Markov chain. The Brownian motion W describes the uncertainty affecting the aggregate consumption, the aggregate dividend, and the agent's information set across growth regimes.

The economy

- There are K regimes indexed by $s_1, \ldots, s_K$, i.e., the set of possible regimes is $\Omega^s = \{s_1, . . ., s_K\}$. The Markov chain takes values $s_t \in \Omega^s$s and has switching intensity given by the matrix $\Lambda = [\lambda_{lk}]$. The entry λ_{lk} is the instantaneous rate of switching from regime s_l to s_k

$$ds_t = \left(\Lambda dt + d\tilde{N}_t\right)' s_t,$$

The evolutions of the aggregate consumption C

- the aggregate dividend D, and an information factor I are driven by (W, s) and described by

$$\frac{dC_t}{C_t} = \mu^C(s_t)dt + \sigma^C dW_t^C,$$

$$\frac{dD_t}{D_t} = \mu^D(s_t)dt + \sigma^D\left(\rho dW_t^C + \sqrt{1-\rho^2}dW_t^G\right),$$

and

$$\frac{dI_t}{I_t} = \mu^I(s_t)dt + \sigma^I\left(\rho^{IC}dW_t^C + \rho^{ID}dW_t^G + \sqrt{1-(\rho^I)^2}dW_t^Y\right)$$

Information and beliefs

- The information structure is the filtration F(·) = {Ft : t ∈ [0, ∞)} generated by the observable processes (C, D, I), where Ft = σ − {(Cs,Ds, Is) : s ∈ [0,t]} is the information at time t.
- Observation of (C, D, Y) is equivalent to the observation of the orthogonalized system

$$\frac{dC_t}{C_t} = \mu^C(s_t)dt + \sigma^C dW_t^C, \qquad (5$$

$$\frac{dG_t}{G_t} = \mu^G(s_t)dt + \sigma^G dW_t^G, \qquad \frac{dY_t}{Y_t} = \mu^Y(s_t)dt + \sigma^Y dW_t^Y$$

beliefs

- Based on initial priors and the information accumulated, the representative agent forms beliefs about the regime s_t.
- the conditional probability pk evolves according to

$$dp_{kt} = p_{kt}\left(\mu^p_{kt}dt + \Delta^C_{kt}d\nu^C_t + \Delta^G_{kt}d\nu^G_t + \Delta^Y_{kt}d\nu^Y_t\right),$$

where $\mu^p_{kt} = \sum_{j=1}^{K} p_{jt}\lambda_{jk}/p_{kt}$, and

$$\Delta^C_{kt} = \frac{\mu^C_k - \widehat{\mu}^C_t}{\sigma^C}, \quad \Delta^G_{kt} = \frac{\mu^G_k - \widehat{\mu}^G_t}{\sigma^G}, \quad \Delta^Y_{kt} = \frac{\mu^Y_k - \widehat{\mu}^Y_t}{\sigma^Y}$$

beliefs

$$d\nu_t = \begin{bmatrix} d\nu_t^C \\ d\nu_t^G \\ d\nu_t^Y \end{bmatrix} = \begin{bmatrix} \frac{1}{\sigma^C} & 0 & 0 \\ 0 & \frac{1}{\sigma^G} & 0 \\ 0 & 0 & \frac{1}{\sigma^Y} \end{bmatrix} \begin{bmatrix} \frac{dC_t}{C_t} - \widehat{\mu}_t^C dt \\ \frac{dG_t}{G_t} - \widehat{\mu}_t^G dt \\ dY_t - \widehat{\mu}_t^Y dt \end{bmatrix}$$

$$= \begin{bmatrix} dW_t^C + \frac{\mu^C(s_t) - \widehat{\mu}_t^C}{\sigma^C} dt \\ dW_t^G + \frac{\mu^G(s_t) - \widehat{\mu}_t^G}{\sigma^G} d \\ dW_t^Y + \frac{\mu^Y(s_t) - \widehat{\mu}_t^Y}{\sigma^Y} dt \end{bmatrix}.$$

Preferences

- Beliefs-dependent utility

$$u(c_t, p_t, t) = \sum_{k=1}^{K} \frac{1}{1-R_k} a_t c_t^{1-R_k} p_{kt},$$

- $u(c_t, p_t, t)$ is the instantaneous utility at time t. The utility function depends on consumption c_t, the conditional probability of the regime p_t, and time t.
- The utility function is increasing and concave with respect to consumption. Marginal utility of consumption satisfies the Inada conditions at zero and ∞.

稻田条件

- 1.f(0)=0；
- 2.一阶导数大于0，二阶导数小于0；
- 3.当自变量趋于0时，一阶导数的极限无穷大
- 4.当自变量趋于无穷大时，一阶导数的极限等于0

Financial markets

- There are two assets: a risky and a riskless asset.
- The riskless asset is a money market account paying interest at the rate r. It is in zero net supply.
- The risky asset is a long-lived claim to aggregate dividends. It evolves according to

$$\frac{dS_t + D_t dt}{S_t} = \mu_t^S dt + \sigma_t^S \left(\rho_t^{SC} d\nu_t^C + \rho_t^{SG} d\nu_t^D + \rho_t^{SY} d\nu_t^Y \right)$$

Interest rate and market prices of risk

- state price density (SPD)
- the equilibrium interest rate is the drift of the state price density
- the market prices of risk the volatility of the state price density.

$$d(y\xi_t) = -(y\xi_t)\left(r_t dt + \theta_t^C d\nu_t^C + \theta_t^G d\nu_t^G + \theta_t^Y d\nu_t^Y\right)$$

state price density (SPD)

- The equilibrium state price density (SPD) is proportional to marginal utility of consumption of the representative agent evaluated at aggregate consumption. That is,u_c $(C_t, p_t, t) = y\xi t$, where $y = u_c$ $(C_0, p_0, 0)$ is the marginalutility at t = 0 and ξ_t is the SPD.

$$d(y\xi_t) = u_{ct}dt + u_{cc}dC_t + \frac{1}{2}u_{ccc}d[C]_t + \sum_k u_{cp_k}dp_{kt} + \frac{1}{2}\sum_{k,j} u_{cp_k p_j}d\big[p_k, p_j\big]_t + \sum_k u_{ccp_k}d[C, p_k]_t.$$

Interest rate and market prices of risk

- The equilibrium interest rate and market prices of risk are given by

$$r_t = \beta + \left(\sum_{k=1}^{K} R_k q_{kt}\right)\widehat{\mu}_t^C - \frac{1}{2}\left(\sum_{k=1}^{K} R_k P_k q_{kt}\right)(\sigma^C)^2 - \sum_{k=1}^{K} \mu_{kt}^p q_{kt} + \sum_{k=1}^{K} R_k q_{kt}(\mu_k^C - \widehat{\mu}_t^C),$$

market prices of risk

$$\theta_t^C = \left(\sum_{k=1}^{K} R_k q_{kt}\right)\sigma^C - \sum_k q_{kt}\,\Delta_{kt}^C, \qquad \theta_t^G = -\sum_k q_{kt}\,\Delta_{kt}^G$$

$$\theta_t^Y = -\sum_k q_{kt}\,\Delta_{kt}^Y$$

Stock price and return volatility

- The equilibrium stock market value is given by the non-linear
- formula

$$S_t = E_t\left[\int_t^\infty \frac{e^{-\beta s}\sum_{k=1}^{K} C_s^{-R_k} p_{ks} D_s}{e^{-\beta t}\sum_{k=1}^{K} C_t^{-R_k} p_{kt}} ds\right] = D_t Z_t' \Upsilon p_t$$

- The volatility of the stock market return is

$$\sigma_t^S = \sqrt{\left(\sigma_t^{SC}\right)^2 + \left(\sigma_t^{SG}\right)^2 + \left(\sigma_t^{SY}\right)^2}$$

$$
\begin{bmatrix} \sigma_t^{SC} \\ \sigma_t^{SG} \\ \sigma_t^{SY} \end{bmatrix} = \begin{bmatrix} \rho\sigma^D + \sigma_t^{SCR} + \sigma_t^{SCG} \\ \sqrt{1-\rho^2}\sigma^D + \sigma_t^{SGG} \\ \sigma_t^{SYG} \end{bmatrix}
$$

$$
\sigma_t^{SCR} = Z_t' \mathrm{diag}_K\left[-R_k\sigma^C\right]\left(\frac{\Upsilon}{Z_t'\Upsilon p_t} - I_K\right)p_t
$$

$$
\sigma_t^{S\alpha G} = Z_t'\left(\frac{\Upsilon}{Z_t'\Upsilon p_t} - I_K\right)\left(\frac{\mathrm{diag}(\bar{\mu}^\alpha) - \widehat{\mu}_t^\alpha I_K}{\sigma^\alpha}\right)p_t
$$

for $\alpha \in \{C, G, Y\}$.

Empirical results

- Data description
- Estimation uses quarterly data from January 1957 to January 2014. The per capita consumptions of nondurable goods (C_n,t) and services (C_s,t) are obtained from the SaintLouis Federal Reserve Bank. Consumption growth is defined as

$$\ln \frac{C_{s,t+1} + C_{n,t+1}}{C_{s,t} + C_{n,t}}$$

dividend series D_t

- Using the CRSP value weighted return indexes including dividends ($vwretd_t$) and excluding dividends ($vwretx_t$) gives the dividend series D_t,

$$P_{t+1} = P_t(1 + \mathrm{vwretx}_{t+1}),$$

$$D_{t+1} = P_{t+1}\left[\frac{(1 + \mathrm{vwretd}_{t+1})}{(1 + \mathrm{vwretx}_{t+1})} - 1\right]$$

PDR and Real returns

- The PDR is obtained by dividing the current price index level by the sum of the 12 previous months' dividends.
- Real returns are obtained by adjusting for inflation using the seasonally adjusted consumer price index

real rate

- Quarterly series of ex-ante real three-month rates and real ten year rates are constructed from monthly series of nominal yields .
- The ex-post real rate is obtained by subtracting the realized inflation from the observed three-month treasury bill rate.It is then regressed against the average quarterly log inflation over the previous year $\pi_{t-12,t}$ .The same procedure is used, with an adjustment for the time period, for the ten-year ex-ante real rate.

$$y_{3,t} - \pi_{t,t+3} = \beta_0 + \beta_1 y_{3,t} + \beta_2 \pi_{t-12,t} + \varepsilon_{t+3}$$

information variable

The information variable in Eq. is defined as the unemployment rate (UE)

- $I_t = UE_t$,

$$\frac{dI_t}{I_t} = \mu^I(s_t)dt + \sigma^I\left(\rho^{IC}dW_t^C + \rho^{ID}dW_t^G + \sqrt{1-(\rho^I)^2}dW_t^Y\right)$$

Parameter estimates

- The set of parameters is partitioned into three subsets

$$\Theta = \Theta_1 \cup \Theta_2 \cup \Theta_3.$$

$$\Theta_1 \equiv \left\{\sigma^C, \sigma^D, \sigma^I, \rho, \rho^{IC}, \rho^{ID}\right\},$$

$$\Theta_2 \equiv \left\{\mu_1^C, \mu_2^C, \mu_3^C, \mu_1^D, \mu_2^D, \mu_3^D, \mu_1^I, \mu_2^I, \mu_3^I, \lambda_{12}, \lambda_{13}, \right.$$
$$\left. \lambda_{21}, \lambda_{23}, \lambda_{31}, \lambda_{32}, R_{\min}, \beta\right\},$$

$$\Theta_3 \equiv \{R_2, R_3\}.$$

Growth regime		
Normal	Low	High
Consumption		
μ_1^C	μ_2^C	μ_3^C
0.01667	0.01241	0.03996
(0.0065058)	(0.0057009)	(0.0083858)
Dividend		
μ_1^D	μ_2^D	μ_3^D
0.00672	0.00591	0.01674
(0.010801)	(0.0075845)	(0.010102)

Growth regime		
Normal	Low	High
Unemployment		
μ_1^{UE}	μ_2^{UE}	μ_3^{UE}
0.00583	0.14717	–0.09679
(0.038985)	(0.043136)	(0.04412)
Preferences: risk aversion		
R_1	R_2	R_3
1.4384	1.9251	1.5938
(0.35991)	(0.04735)	(0.00962)

Preferences: subjective discount rate

β_1	β_2	β_3
0.00669	0.00669	0.00669
(0.0018092)	(0.0018092)	(0.0018092)

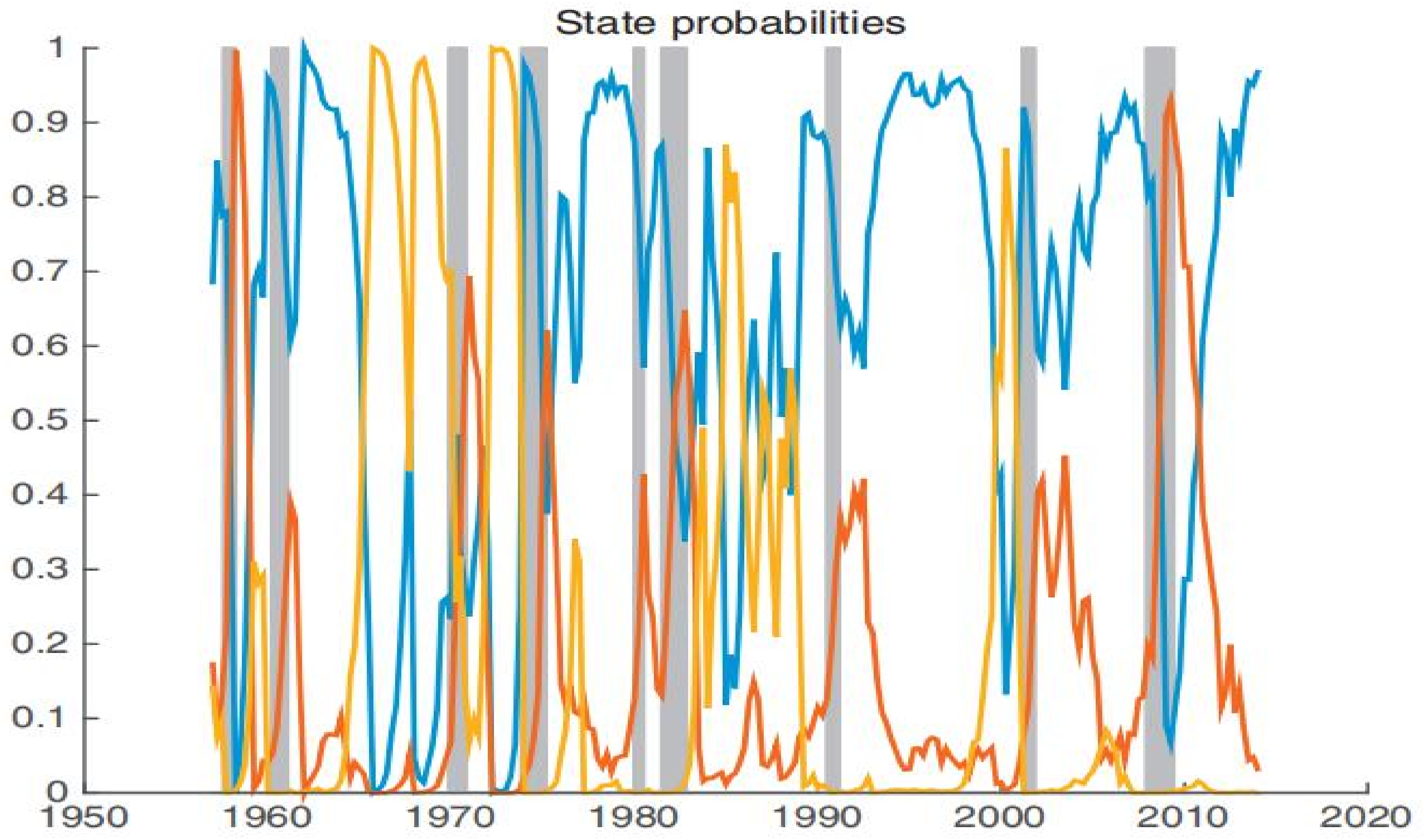
Standard deviations and correlations

	Consumption	Dividend	Unemployment
Consumption	0.0092	0.1664	–0.3914
	(0.0006)	(0.0653)	(0.0594)
Dividend	0.1664	0.0473	–0.3231
	(0.0653)	(0.0079)	(0.0596)
Unemployment	–0.3914	–0.3231	0.1244
	(0.0594)	(0.0596)	(0.0137)

	Infinitesimal generator			
	Normal	Low	High	Steady state probabilities
Normal	–0.0566708103	0.0566705914	0.0000002189	0.6859
	-	(0.015333)	(1.8779e–06)	
Low	0.2130196298	–0.2249243329	0.0119047031	0.1728
	(0.032514)	-	(0.00365)	
High	0.0145535959	0.0000026559	–0.0145562519	0.1413
	(0.0051444)	(1.8348e–06)	-	

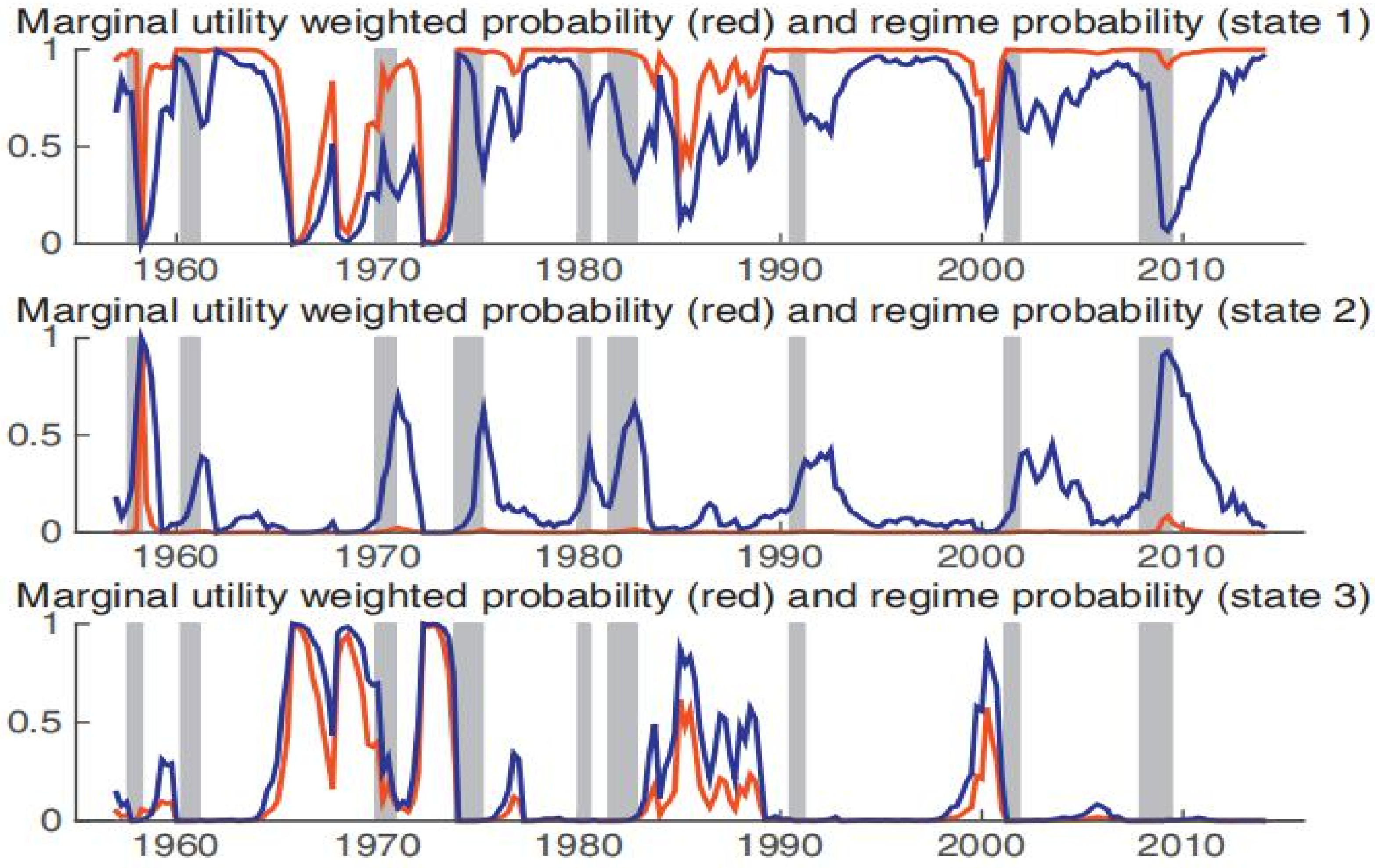
Time series implications

- Latent factors dynamics
- The plot shows the probabilities of state 1 (normal – blue), state 2 (recession – red) and state 3 (boom – yellow). Parameters are estimates from steady state probabilities. Risk aversion parameters are R_1 = 1.4384, R_2 = 1.9251, and R_3 = 1.5938. Conditional probabilities are updated using innovations from consumption, dividend, and unemployment time series. Grey columns are recessions identified by the NBER.（返回PPT18）
- state 1 as the normal regime, state 2 as the recession regime, and state 3 as the expansion/boom regime


State probabilities
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1950
1960
1970
1980
1990
2000
2010
2020

marginal utility weighted probabilities (MUWP)

- The plots shows marginal utility weighted probabilities (MUWP) (red) and physical probabilities (blue) for regime 1 (top panel), 2 (middle panel),and 3 (bottom panel). Risk aversion parameters are R1 =1.4384, R2 = 1.9251, and R3 = 1.5938. Conditional probabilities are updated using innovations from consumption, dividend, and unemployment time series. Grey columns are recessions identified by the NBER



Marginal utility weighted probability (red) and regime probability (state 1)
Marginal utility weighted probability (red) and regime probability (state 2)
Marginal utility weighted probability (red) and regime probability (state 3)
0
0.5
1
1960
1970
1980
1990
2000
2010

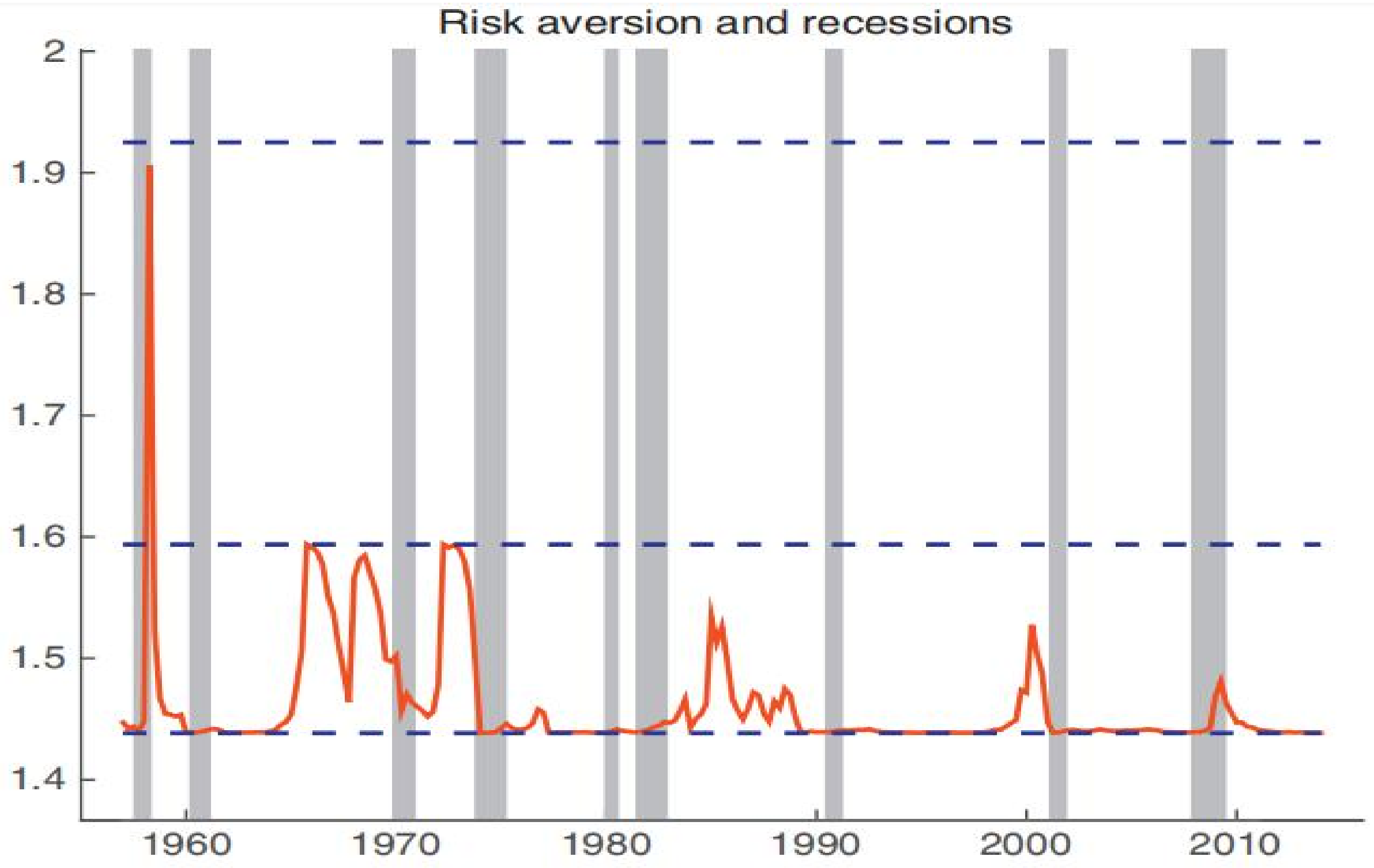
Risk aversion

- The plot shows the model risk aversion (red) and the intra-regime risk aversions R_1, R_2 (dashed blue). Risk aversion parameters are R1 =1.4384,R2 = 1.9251, and R3 = 1.5938. Conditional probabilities are updated using innovations from consumption, dividend, and unemployment time series.

$$R_t = -\frac{u_{cc} c_t}{u_c} = \sum_{k=1}^{K} R_k q_{kt},$$

where

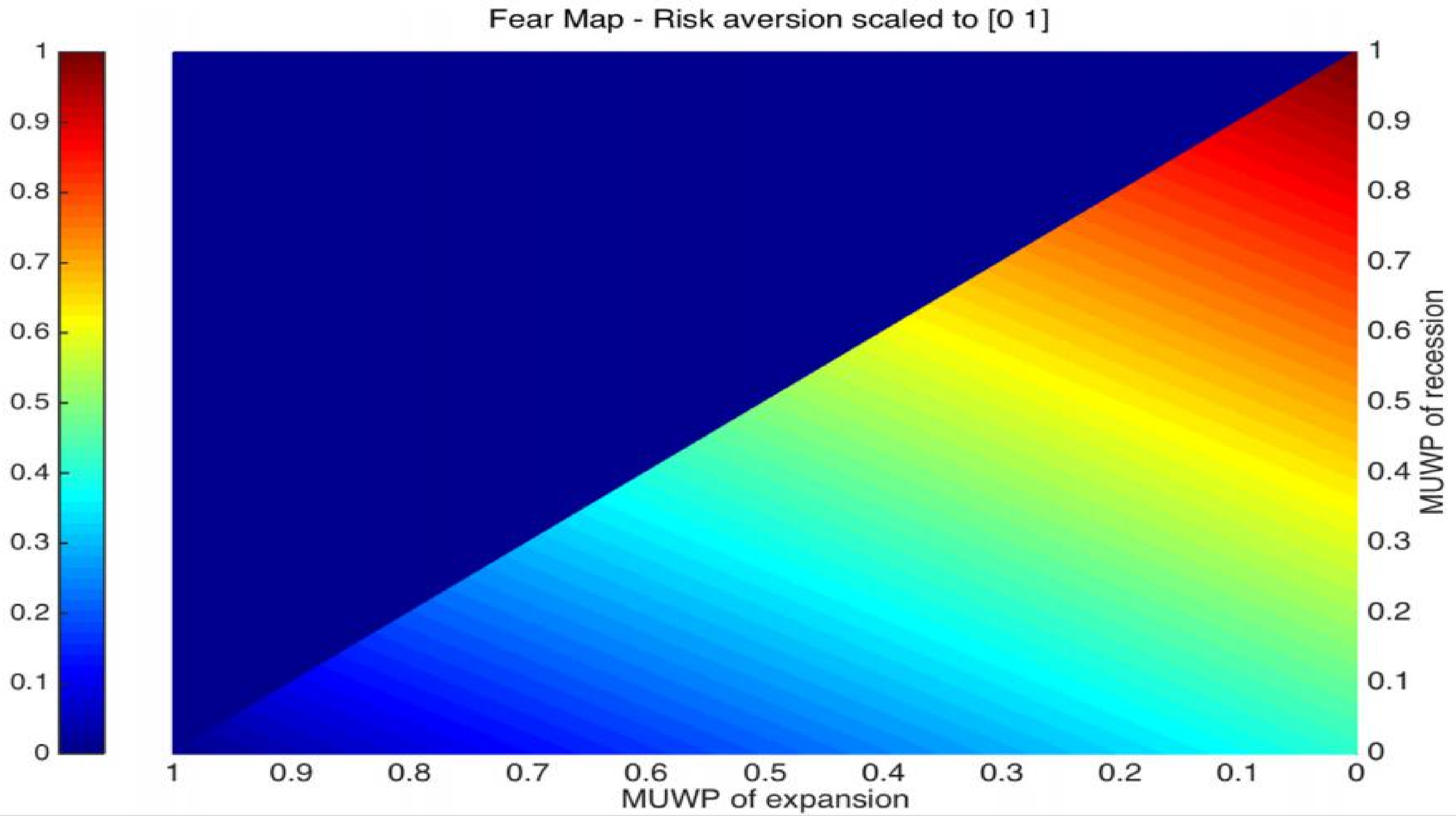
$$q_{kt} = \frac{c_t^{-R_k} p_{kt}}{\sum_{k=1}^{K} c_t^{-R_k} p_{kt}}.$$



Risk aversion and recessions
2
1.9
1.8
1.7
1.6
1.5
1.4
1960
1970
1980
1990
2000
2010

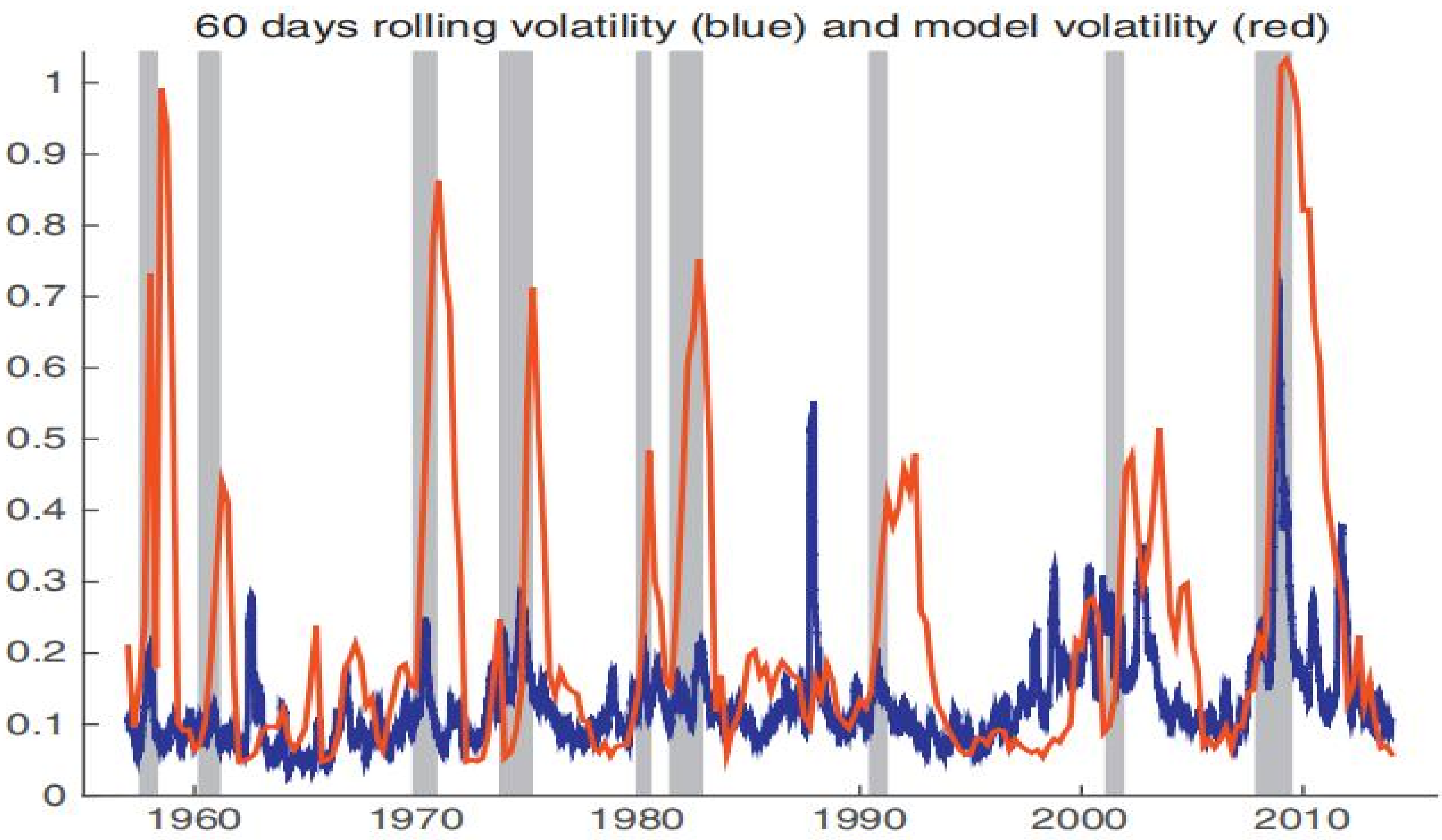
emotions

- The behavior of risk aversion reflects the effects of emotions. Later stages of booms are often associated with apprehension about the future
- i.e., fear of a crash, driving investors to contemplate liquidating their positions.
- The plot shows the model risk aversion as a function of the MUWP of recession and expansion. Risk aversions R_j are scaled to the interval [0,1] following $(R_j - Rmin)/(Rmax - Rmin)$.


Fear Map - Risk aversion scaled to [0 1]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MUWP of expansion
MUWP of recession

Return volatility and variance

- The plot shows model volatility (red) and realized volatility (blue). Realized volatility is calculated using a 60-day rolling window. Parameters are estimates from steady state probabilities
- Overall, the model reflects the countercyclical nature of excess return volatility found in the data.
- Model spikes coincide with spikes in the probability of the recession regime and the simultaneous drop in the probability of the normal regime


60 days rolling volatility (blue) and model volatility (red)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1960
1970
1980
1990
2000
2010

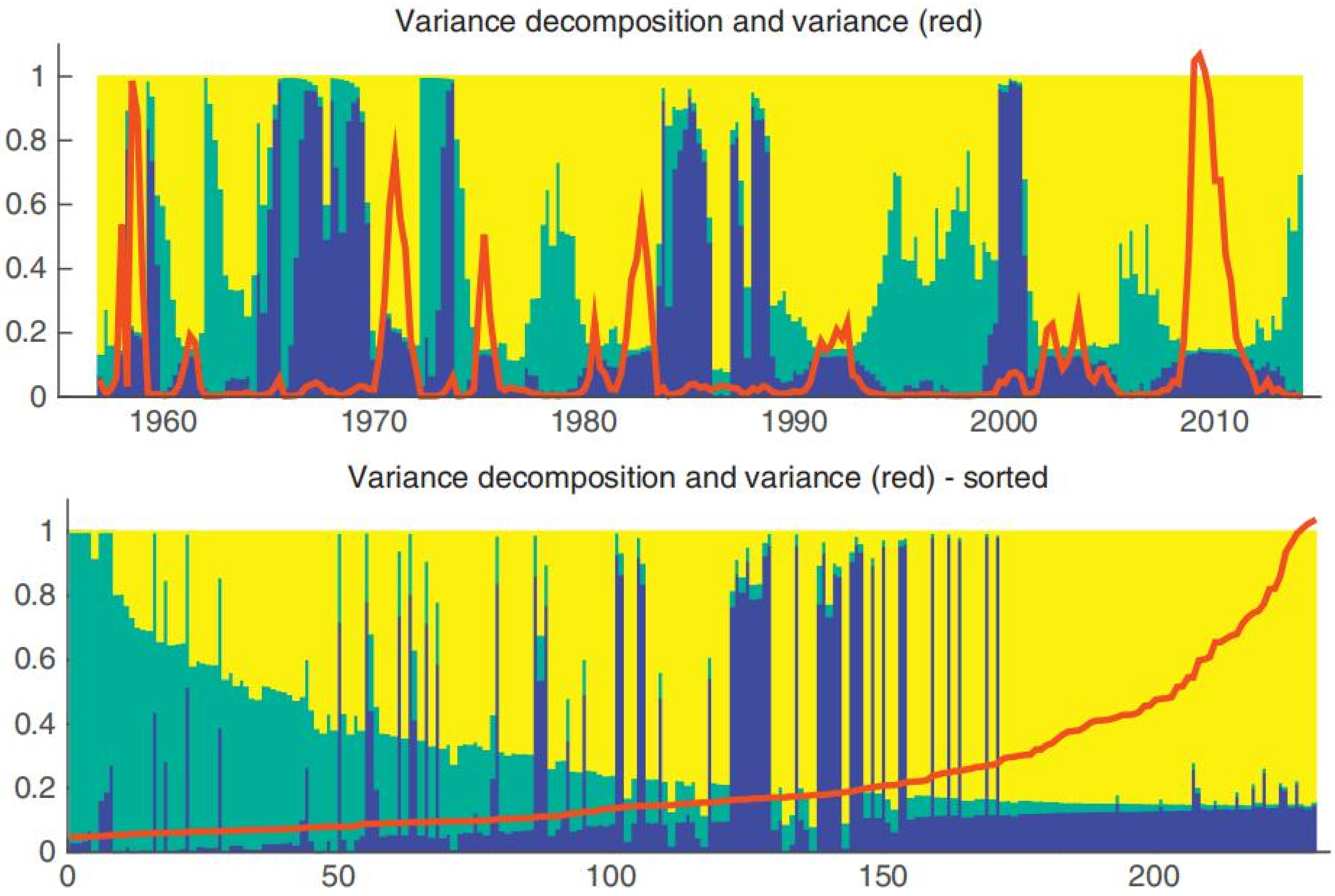
Return volatility and variance

- To understand the sources of volatility, it is useful to focus on the variance of excess returns,

$$(\sigma_t^S)^2 = (\sigma_t^{SC})^2 + (\sigma_t^{SG})^2 + (\sigma_t^{SY})^2$$

Return volatility and variance

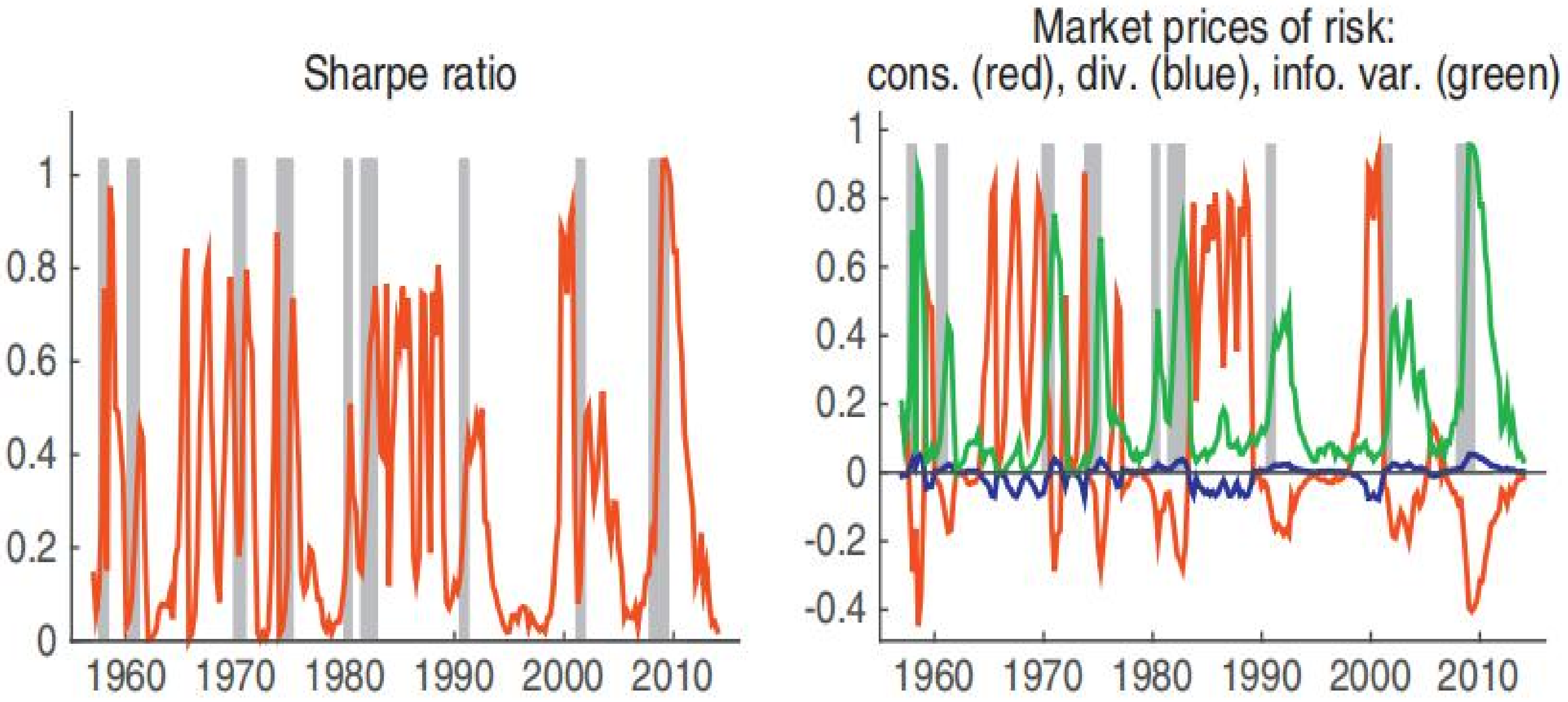
- The plot shows the stock return variance decomposition along with the model implied stock variance over time in the top panel and displayed by increasing level of variance in the bottom panel. The three components correspond to consumption source (blue),dividend source (green), and information source (yellow).



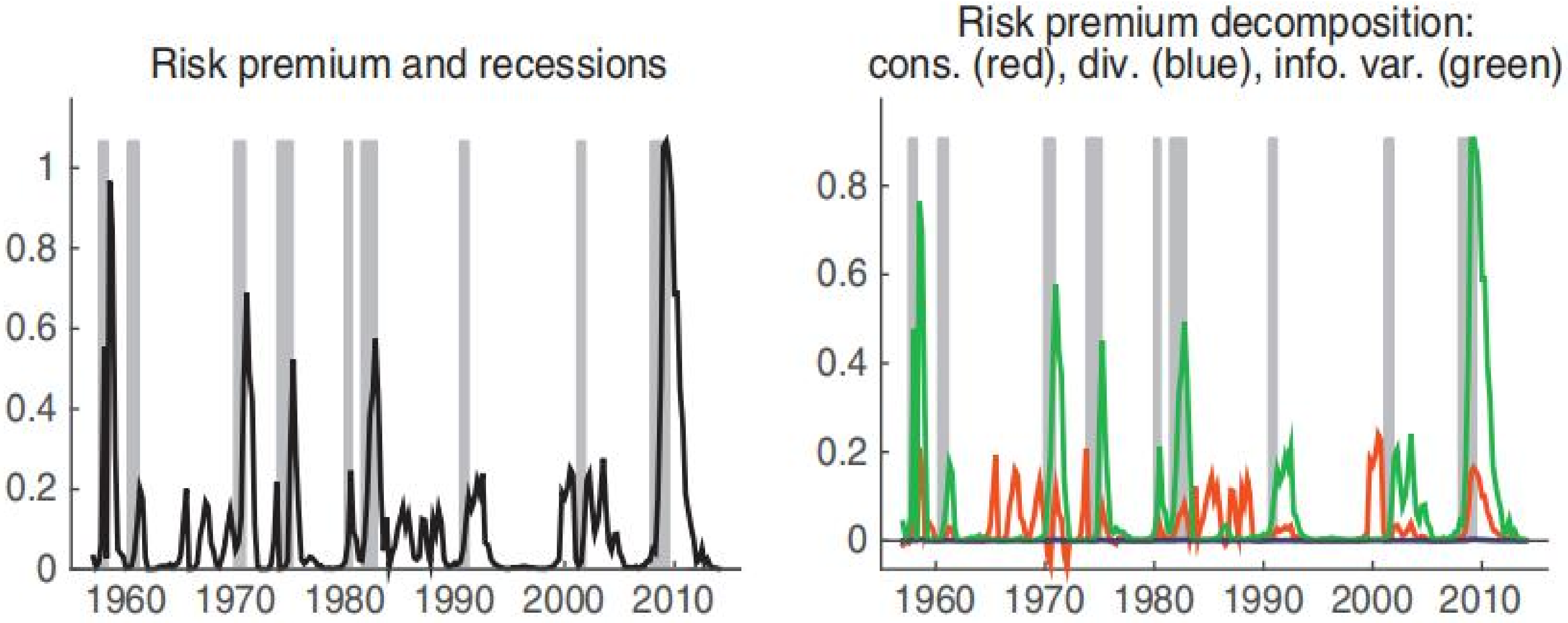
Variance decomposition and variance (red)
1
0.8
0.6
0.4
0.2
0
1960
1970
1980
1990
2000
2010
Variance decomposition and variance (red) - sorted
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200

Sharpe ratio and equity premium

- It tends to be countercyclical in normal and recession regimes, but becomes procyclical in expansions.Overall, it is heavily influenced by the behaviors of the consumption and information factor components.



Sharpe ratio
1
0.8
0.6
0.4
0.2
0
1960
1970
1980
1990
2000
2010
Market prices of risk:
cons. (red), div. (blue), info. var. (green)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1960
1970
1980
1990
2000
2010



Risk premium and recessions
1
0.8
0.6
0.4
0.2
0
1960
1970
1980
1990
2000
2010
Risk premium decomposition:
cons. (red), div. (blue), info. var. (green)
0.8
0.6
0.4
0.2
0
1960
1970
1980
1990
2000
2010

Return predictability

- Specifically, define the information risk premium (IRP) as

$$\mathrm{IRP_t} \equiv \sigma_t^S \rho_t^{SY} \theta_t^Y$$

- It is specific to our model and arises because marginal utility depends on beliefs, and because information in Y is used to update posterior beliefs.

regression

- The following regression at quarterly frequency with a maximum of 20 lags (hence five years),

$$r^e_{t+k} = \mathrm{a} + \mathrm{b}_{\mathrm{IRP}}\mathrm{IRP}_\mathrm{t} + \mathrm{b}_{\mathrm{Div.Yield}}\log[\mathrm{Div.Yield}]_t + \mathrm{b}_{\mathrm{CAY}}\mathrm{CAY}_t + \varepsilon_{t+k},$$

- 谢谢大家
- 恳请老师和伙伴们提出宝贵建议