Mitigating Disaster Risks to Sustain Growth

Harrison Hong   Neng Wang   Jinqiang Yang

汇报人：任宇卓

2022.5.4

Introduction

- The arrival of natural disasters can cause not only significant human suffering but also prolonged economic damage.

  - weather disasters： hurricanes
  - pandemics： 1918 Flu、Covid-19

Introduction

“Mitigation is the effort to reduce loss of life and property by lessening the impact of disasters. In order for mitigation to be effective we need to take action now—before the next disaster—to reduce human and financial consequences later (analyzing risk, reducing risk, and insuring against risk).”

---FEMA website

The extent of damages depends on investment and greater preparedness, be it greater environmental protection in the case of climate disasters or better public health capabilities in the case of pandemics.

---World Health Organization (2018)

1. Cost and Benefit of Mitigation
2. belief---preparedness---damage function
3. market

Abstract

- We provide the planner’s solution to a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate future damages. Mitigation cannot be decentralized due to positive externalities from curtailing aggregate risks. First-best can be implemented by capital taxes and mitigation subsidies.
- Willingness-to-pay, toward public health for pandemics or environmental protection for climate disasters, depends on mitigation efficacy. Efficacy can be inferred from damage functions that depend on prior arrivals which determine preparedness.
- Regulatory risks arise since disaster leads to pessimistic arrival-rate beliefs and taxes or mandates to fund mitigation, which reduce consumption, investment and stock-market value.

Model： Production, Capital Dynamics, and Disasters

- ✓ Aggregate output：

$$Y_t = AK_t \, , \tag{1}$$

- ✓ Aggregate resource constraint:

$$Y_t = C_t + (I_t + \Phi_t) + X_t \, . \tag{2}$$

$$\Phi(I, K) = \phi(i)K$$

$c_t = C_t/K_t$, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$.

Model： Capital Accumulation

✓ capital stock evolves as:

$$dK_t = I_{t-}dt + \sigma K_{t-}d\mathcal{W}_t - (1 - Z)K_{t-}d\mathcal{J}_t \ . \tag{3}$$

- When a jump arrives ($dJ_t = 1$)， capital stock changes from K_{t-} to ZK_{t-} , Z is the recovery fraction.

- $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, Z, conditional on a jump arrival.

Model： arrival rate of the jump process

- Two possible state: good state (G) and bad state (B).

  - jump arrival rate in G is λ_G , jump arrival rate in B is λ_B. Naturally, $\lambda_B > \lambda_G$.
  - While the state is constant over time, the household does not observe the state and therefore has to learn about the value of λ over time to assess the likelihood that the arrival rate is high or low.

Model: Mitigation Technology and Payoffs

- Distribution for the recovery fraction Z at t conditional on a jump arrival depends on the pre-jump mitigation spending x_{t-} .

  - The distribution of the Post-jump fractional recovery Z changes from $\Xi(Z)$ to $\Xi(Z, x_{t-})$, the density function changes from $\xi(Z)$ to $\xi(Z, x_t)$.
  - If mitigation spending X doubles, the benefit of mitigation also doubles.
  - The domain for the admissible values of Z is $(0, \bar{Z})$, where $0 < \bar{Z} < 1$ is a constant.

Preferences

➢ Representative consumer has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f(C_s, V_s) ds \right] , \tag{6}$$

where $f(C, V)$ is known as the normalized aggregator given by

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} . \tag{7}$$

- ρ : the rate of time preference
- ψ : the elasticity of intertemporal substitution (EIS)
- γ : the coefficient of relative risk aversion,

$\omega = (1 - \psi^{-1})/(1 - \gamma)$

Solution： Learing

Learning. The household dynamically updates her belief about the arrival rate of disasters. Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$. That is,

$$\pi_t = \mathbb{P}(\lambda_t = \lambda_B | \mathcal{F}_t)\,, \tag{9}$$

where $\mathcal{F}_t$ is the household's information set up to t. At time t, the expected jump arrival rate, denoted by λ_t, is given by

$$\lambda_t = \lambda(\pi_t) = \lambda_B \pi_t + \lambda_G (1 - \pi_t)\,, \tag{10}$$

Mathematically, the household updates her belief by following the Bayes rule:[12]

$$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-} dt\right)\,, \tag{11}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B \pi + \lambda_G (1-\pi)} > 0\,. \tag{12}$$

Solution： Learing

ematically, if $d\mathcal{J}_t = 1$, we have $d\pi_t = \mu_\pi(\pi_{t-})dt$, where

$$\mu_\pi(\pi) = -\sigma_\pi(\pi)\lambda(\pi) = \pi(1-\pi)(\lambda_G - \lambda_B) < 0\,. \tag{14}$$

Now suppose that there is no jump during a finite time interval (s,t), i.e., $dJ_v = 0$ for $s < v \leq t$. By using (14) to integrate π from s to t conditional on no jump, we obtain the following logistic function:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B-\lambda_G)(t-s)}}{1+\pi_s(e^{-(\lambda_B-\lambda_G)(t-s)}-1)}\,. \tag{15}$$

lief process π is a martingale. When a disaster strikes at t, the household's belief immediately increases by $\sigma_\pi(\pi)$ from the pre-jump level π to $\pi^{\mathcal{J}}$, where

$$\pi^{\mathcal{J}} = \pi + \sigma_\pi(\pi) = \frac{\pi\lambda_B}{\lambda(\pi)} > \pi\,. \tag{13}$$

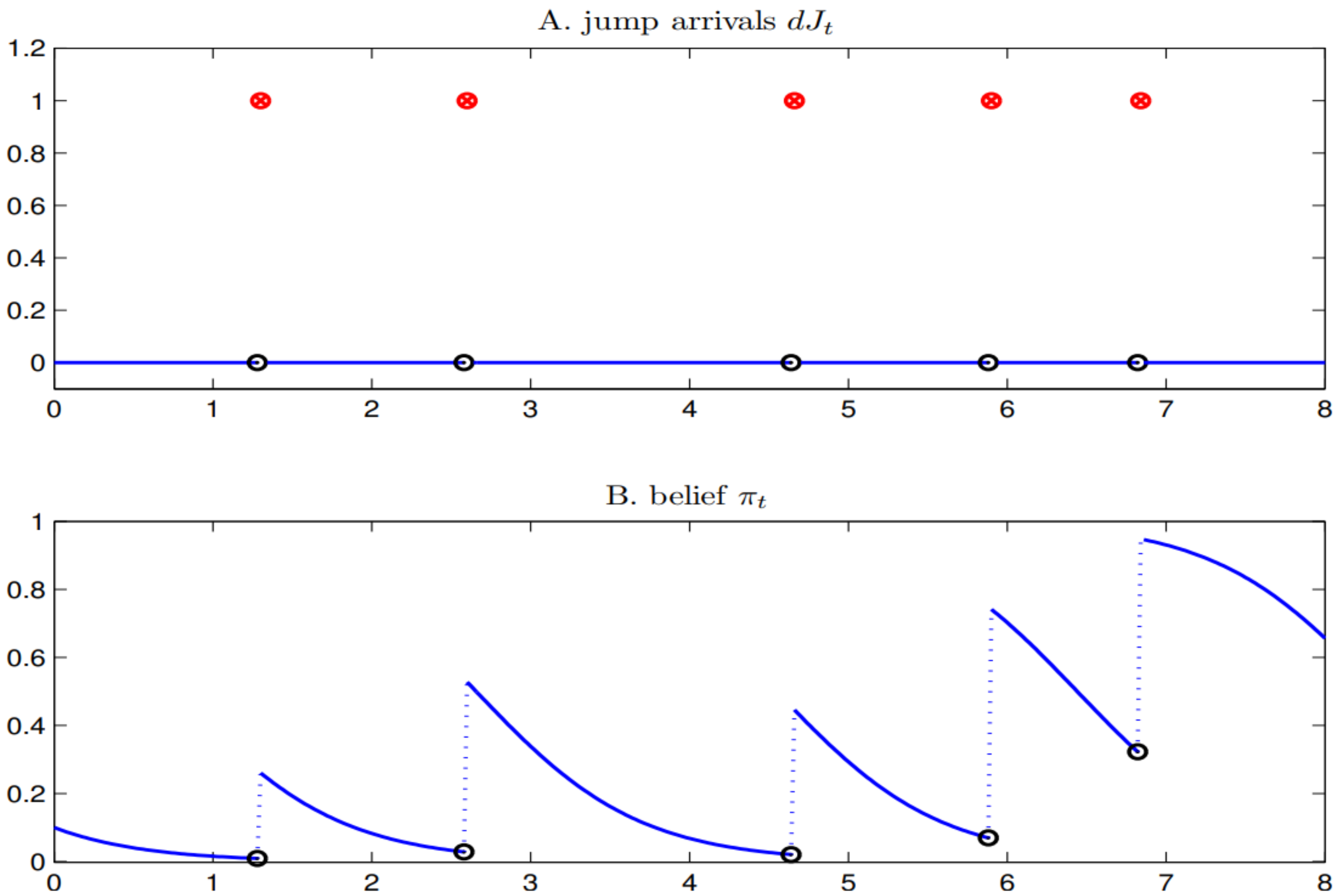


Figure 1: This figure simulates a path for jump arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.1$.

Planner's Optimization

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right]$$

- The social planner maximizes the representative household's utility given in (6)-(7) subject to the capital accumulation and the aggregate resource constraint.

$$dK_t = I_{t-} dt + \sigma K_{t-} d\mathcal{W}_t - (1 - Z) K_{t-} d\mathcal{J}_t \qquad Y_t = C_t + (I_t + \Phi_t) + X_t \,.$$

Let $V(K, \pi)$ denote the value function.

Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} 0 \;=\; & \max_{C, I, x} f(C, V) + I V_K(K, \pi) + \mu_\pi(\pi) V_\pi(K, \pi) + \frac{1}{2} \sigma^2 K^2 V_{KK}(K, \pi) \\ & + \lambda(\pi) \mathbb{E} \left[V\left(ZK, \pi^{\mathcal{J}} \right) - V(K, \pi) \right] \,, \end{aligned} \tag{16}$$

- FOC for investment I：

$$(1 + \Phi_I(I, K)) f_C(C, V) = V_K(K, \pi) \,. \tag{17}$$

- FOC with respect to mitigation is

$$K f_C(C, V) = \lambda(\pi) \int_0^{\overline{Z}} \left[\frac{\partial \xi(Z; x)}{\partial x} V\left(ZK, \frac{\pi \lambda_B}{\lambda(\pi)} \right) \right] dZ \,, \tag{18}$$

$$V(K,\pi) = \frac{1}{1-\gamma}\left(b(\pi)K\right)^{1-\gamma}, \tag{19}$$

Using the FOCs (17) and (18) and substituting the value function $V(K,\pi)$ given in (19) together with the implied policy rules into the HJB equation (16), and simplifying the equations, we obtain the following three-equation ODE system for $b(\pi)$, $i(\pi)$, and $x(\pi)$:

$$\begin{aligned} 0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi} - 1\right] + i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\ &\quad + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right], \end{aligned} \tag{20}$$

$$b(\pi) = \left[A - i(\pi) - \phi(i(\pi)) - x(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}, \tag{21}$$

$$1 = \frac{\lambda(\pi)(1+\phi'(i(\pi)))}{1-\gamma}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x(\pi))}{\partial x}Z^{1-\gamma}\right]dZ\,. \tag{22}$$

boundary conditions at $\pi = 0$ and $\pi = 1$.

Expected Fractional Loss, Growth Rate

- The cdf of Z is given by the following power function :

$$\Xi(Z;x) = \left(Z/\overline{Z}\right)^{\beta(x)} \;;\; 0 \le Z \le \overline{Z} \;, \tag{26}$$

- Conditional on a jump arrival, the expected fractional capital loss :

$$\ell(\pi) = 1 - \mathbb{E}(Z) = 1 - \frac{\beta(x(\pi))}{\beta(x(\pi)) + 1}\overline{Z}\,. \tag{27}$$

- Expected growth rate:

$$g(\pi) = i(\pi) - \lambda(\pi)\ell(\pi) = i(\pi) - \lambda(\pi)\left(1 - \frac{\beta(x(\pi))}{\beta(x(\pi)) + 1}\overline{Z}\right)\,. \tag{28}$$

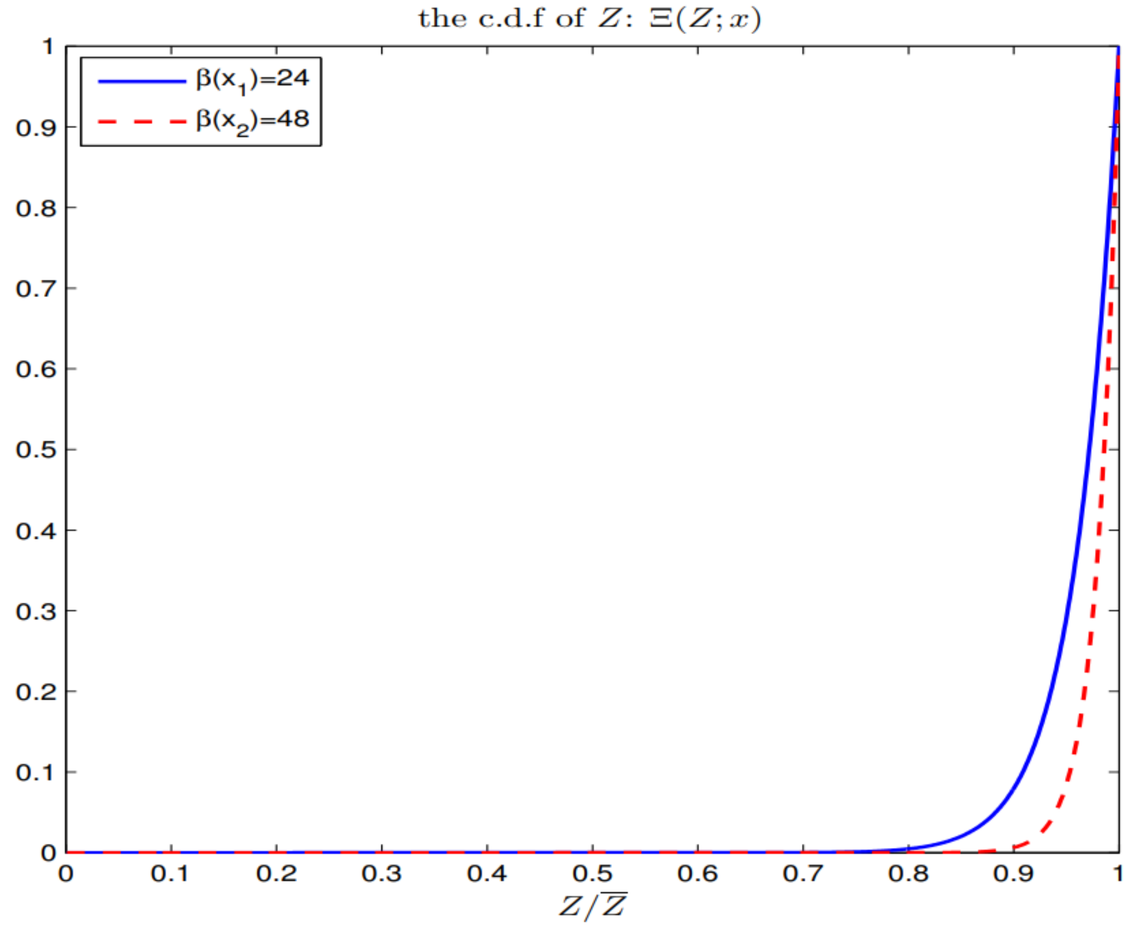


Figure 2: The c.d.f of Z, $\Xi(Z;x)$ for two levels of $\beta(x)$. In this plot, as $\beta(x_1) = 24$ and $\beta(x_2) = 48$, $\Xi(Z;x_2)$ first-order stochastically dominates $\Xi(Z;x_1)$.

Planner's Value Function: No Mitigation Technology.

- Value function:

$$\widehat{V}(K,\pi)=\frac{1}{1-\gamma}\left(\widehat{b}(\pi)K\right)^{1-\gamma}, \tag{30}$$

where $\widehat{b}(\pi)$ is a measure of welfare (proportional to the certainty equivalent wealth).

$$\begin{aligned}
0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi}-1\right]+i(\pi)-\frac{\gamma\sigma^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\
&\quad +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})-1\right], && (20)\\
b(\pi) &= \left[A-i(\pi)-\phi(i(\pi))-x(\pi)\right]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}, && (21)\\
1 &= \frac{\lambda(\pi)(1+\phi'(i(\pi)))}{1-\gamma}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x(\pi))}{\partial x}Z^{1-\gamma}\right]dZ\,. && (22)
\end{aligned}$$

Cost and Benefit of Mitigation

- Willingness to pay (WTP)

$$V((1-\zeta(\pi))K,\pi)=\widehat{V}(K,\pi)\ . \tag{45}$$

$$V(K,\pi)=\frac{1}{1-\gamma}(b(\pi)K)^{1-\gamma}\,,\qquad \widehat{V}(K,\pi)=\frac{1}{1-\gamma}\left(\widehat{b}(\pi)K\right)^{1-\gamma}$$

we obtain the following equation for $\zeta(\pi)$:

$$\zeta(\pi)=1-\frac{\widehat{b}(\pi)}{b(\pi)}>0\ . \tag{46}$$

Competitive Equilibrium and Market Failure

Disaster Risk Insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z; x^*)dZ$, where x^* is the aggregate (scaled) mitigation spending, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contracts, e.g., the insurance premium payment $p(Z; x^*)$, are priced at actuarially fairly so that investors earn zero profits. $p(Z; x^*)$ depends on not only Z but also x^*. This is because the aggregate mitigation spending x^* changes the distribution for $\Xi(Z)$.

Competitive Equilibrium and Market Failure

We define the recursive competitive equilibrium as follows: (1) The representative household chooses consumption C, allocation to the stock market H, various DIS claims $\delta(Z)$, and mitigation spending X_c to maximize utility as given by (6)-(7). (2) The representative firm chooses investment I and mitigation spending X_f to maximize its market value, which is the present discounted value of future cash flows. Private agents take the equilibrium prices of all goods and financial assets including the risk-free rate $r(\pi)$ and the stock-market price process as given. (3) All markets clear.

Competitive Equilibrium and Market Failure

The representative firm solves the following value maximization problem:[14]

$$\max_{I, X_f} \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0}\left(AK_s - I_s - \Phi_s - X_{f,s}\right) ds\right], \tag{31}$$

HJB equation:

$$\begin{aligned} 0 = \max_{i, x_f} \; & A - i - \phi(i) - x_f - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi) \\ & - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}} Z^{-\gamma}\xi(Z; x^*)dZ - 1\right)\right] q(\pi) \\ & + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}} Z^{1-\gamma}\xi(Z; x^*)dZ - 1\right] q(\pi). \end{aligned} \tag{32}$$

FOC for investment :

$$q(\pi) = 1 + \phi'(i(\pi)), \tag{33}$$

- The household's value function:

$$J(W,\pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}, \tag{34}$$

When a disaster occurs at time t, wealth changes discretely from W_{t-} to $W_t^{\mathcal{J}}$, where

$$W_t^{\mathcal{J}} = W_{t-} - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)H_{t-} + \delta_{t-}(Z)W_{t-}\,.$$

The household accumulates wealth as:

$$\begin{aligned} dW_t &= r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_{t-}) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt - X_{c,t-}dt \\ &\quad - \left(\int_0^{\overline{Z}} \delta_{t-}(Z)p(Z;x_{t-}^*)dZ\right)W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)H_{t-}d\mathcal{J}_t\,. \end{aligned} \tag{B.17}$$

where $u(\pi)$ is to be determined. The household solves the following problem:

$$0 = \max_{c,h,\delta,x_c} \frac{\rho\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - \rho}{1-\psi^{-1}} + \left[r(\pi) - \int_0^{\overline{Z}} \delta(Z)p(Z;x^*)dZ + \frac{(\mu_Q(\pi) - r(\pi))h - c - x_c}{w}\right]$$
$$+ \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} + \lambda(\pi)\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left(\frac{w^{\mathcal{J}}}{w}\right)^{1-\gamma}\xi(Z;x^*)dZ - 1\right], \quad (35)$$

The consumption FOC implied by (35) yields the following consumption rule:

$$c(\pi) = \rho^{\psi}u(\pi)^{1-\psi}w\,. \quad (36)$$

The private sector has no incentives to spend on mitigation:

$$x_c = x_f = 0\,. \quad (37)$$

market equilibrium features no aggregate mitigation spending: $x^* = x_c^* + x_f^* = 0$.

Proposition 2 *There is no mitigation in competitive equilibrium. The competitive equilibrium solution corresponds to the social planner's solution only when there is no mitigation technology (i.e.* $\beta_1 = 0$*):* $\widehat{V}(K_t, \pi_t) = J(W_t, \pi_t)$*, where* $W_t = q(\pi_t)K_t$.

Taxes, Subsidies, and Markets

- The government chooses the optimal path of mitigation spending (financed by time-varying lump-sum taxes) to maximize the household's welfare.

As in Section B, the household accumulates wealth as:

$$dW_t = r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_t) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt \tag{C.35}$$
$$- \left(\int_0^{\overline{Z}} \delta_{t-}(Z)p(Z; x^*_{t-})dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi^{\mathcal{J}}_{t-})}{q^*(\pi_{t-})}\right) H_{t-}d\mathcal{J}_t\,,$$

where x^* is chosen by the government. As it is in the household's interest to choose no mitigation spending, we leave this term out of (C.35). The HJB equation for the household in this setting is:

$$0 = \max_{C,H}\ f(C,J) + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^{\overline{Z}} \delta(Z)p(Z;x^*)dZ\right) W - C\right] J_W$$
$$+ \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\int_0^{\overline{Z}} \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W,\pi)\right] \xi(Z;x^*)dZ\,, \tag{C.36}$$

Imposing the equilibrium outcome on the households' side, we obtain (D.73), (B.27), and (B.28). Using (34) and these conditions to simplify (C.36), we obtain the following ODE for $u(\pi)$:

$$\begin{aligned} 0 \;=\; & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi}-1\right]+\left(\mu_Q(\pi)-\rho^{\psi}u(\pi)^{1-\psi}\right)-\frac{\gamma\sigma^2}{2} \\ & +\mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)}+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})-1\right]. \end{aligned} \tag{C.37}$$

Taking the SDF in (B.23) as given, the firm chooses investment I to solve:

$$\max_{I,X_f}\mathbb{E}\left[\int_0^\infty\frac{\mathbb{M}_t}{\mathbb{M}_0}\left(AK_t-I_t-\Phi_t-X_{f,t}-X_t^*\right)dt\right], \tag{C.38}$$

where X^* is chosen by the government and hence exogenous to the firm. By applying Ito's Lemma to $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X^*_{t-})dt + d\left(\mathbb{M}_t Q_t\right)$, which is a martingale due to no arbitrage (Duffie, 2001), we obtain the following ODE for $q_t = Q_t/K_t = q(\pi_t)$:

$$\begin{aligned} r(\pi)q(\pi) = {} & A - i - \phi(i) - x^* + i(\pi)q(\pi) + \mu_\pi(\pi)q'(\pi) \\ & - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}} Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right]q(\pi) \\ & + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}} Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right]q(\pi)\,. \end{aligned} \tag{C.39}$$

By differentiating (C.39) with respect to i, we obtain the investment FOC:

$$q(\pi) = 1 + \phi'(i)\,. \tag{C.40}$$

By using the aggregate resource constraint, $c^*(\pi) = A - i^*(\pi) - \phi(i^*(\pi)) - x^*$, we obtain the following expression for the equilibrium expected return of the aggregate stock market:

$$\mu_Q(\pi) = \frac{A - i^*(\pi) - \phi(i^*(\pi)) - x^*}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)}\,. \tag{C.41}$$

Taxes, Subsidies, and Markets

- The government chooses the optimal path of mitigation spending (financed by time-varying lump-sum taxes) to maximize the household's welfare.

HJB equation:

$$0 = \max_x \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + \frac{A - i^*(\pi) - \phi(i^*(\pi)) - x}{q^*(\pi)} + i^*(\pi) - \rho^{\psi} u(\pi)^{1-\psi} - \frac{\gamma\sigma^2}{2}$$
$$+ \mu_\pi(\pi)\left(\frac{(q^*(\pi))'}{q^*(\pi)} + \frac{u'(\pi)}{u(\pi)}\right) + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right], \quad (38)$$

The FOC for x :

$$1 = \frac{\lambda(\pi)q^*(\pi)}{1-\gamma}\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\int_0^{\overline{Z}}\left[\frac{\partial\xi(Z;x)}{\partial x}Z^{1-\gamma}\right]dZ\,. \quad (39)$$

$$J(W,\pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}, \qquad V(K,\pi) = \frac{1}{1-\gamma}(b(\pi)K)^{1-\gamma},$$

We obtain

$$u(\pi)q(\pi) = b(\pi)\,, \quad (40)$$

Taxation and Subsidy: Resurrecting First Best

Given the government taxation and subsidy policy, each firm solves the following problem:

$$\max_{I, X_f} \mathbb{E}\left[\int_0^\infty \left(\frac{\mathbb{M}_s}{\mathbb{M}_0}\left(AK_s - I_s - \Phi_s - X_{f,s} - \nu_s K_s\right) + p_{0,s} X_{f,s}\right) ds\right], \tag{42}$$

where $p_{0,s}$ is the time-0 value of the government subsidy to the firm for a unit of its mitigation spending at s for each sample path (e.g., state). The firm makes a tax payment $\nu_s K_s$ and receives a subsidy $p_{0,s}$ for each unit of mitigation spending. Because markets are complete,

The firm's HJB equation is then

$$\begin{aligned} 0 = \max_i\ & (A - \nu(\pi)) - i - \phi(i) - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi) q'(\pi) \\ & - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^{\overline{Z}} Z^{-\gamma}\xi(Z; x^*)dZ - 1\right)\right] q(\pi) \\ & + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^{\overline{Z}} Z^{1-\gamma}\xi(Z; x^*)dZ - 1\right] q(\pi)\,. \end{aligned} \tag{43}$$

The firm's investment FOC is still given by (33). The household optimization is essentially the same as that discussed in Section 4. For brevity, we leave the details out.

Proposition 3 *In a competitive (and complete) market economy, household consumption and corporate investment attain the first-best solution as the planner does in Section 3, provided that the government is benevolent, in the sense that it optimally chooses mitigation spending to maximize household welfare. Alternatively, effective government taxation and subsidies policies can also attain the first-best outcome.*

Calibration Exercise and Parameter Choices

For our quantitative analysis, we use the following linear specification for $\beta(x)$:

$$\beta(x) = \beta_0 + \beta_1 x \,, \tag{29}$$

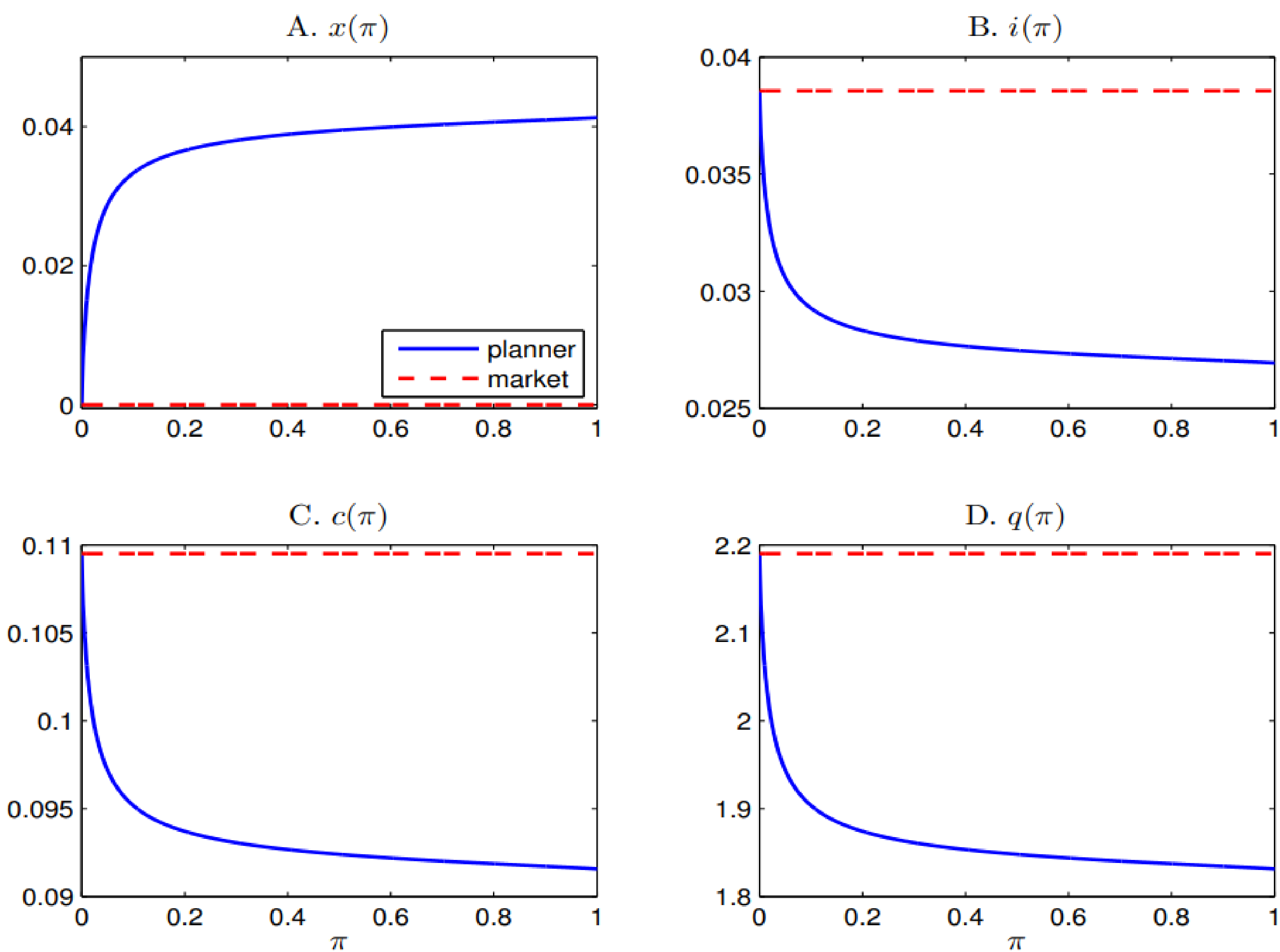
with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for recovery Z in the absence of mitigation. The coefficient β_1 is the elasticity of cdf $\Xi(Z)$ with respect

As in the q literature, e.g., Hayashi (1982), we use a quadratic function:

$$\phi(i) = \frac{\theta i^2}{2} \,, \tag{44}$$

Table 1: PARAMETER VALUES

Parameters	Symbol	Value
time rate of preference	ρ	5%
elasticity of intertemporal substitution	ψ	1
maximal post-jump recovery fraction	$\overline{Z}$	98.1%
power law exponent with no mitigation	β_0	24
jump arrival rate if State is G	λ_G	0.25
coefficient of relative risk aversion	γ	3.1
productivity	A	17.1%
quadratic adjustment cost parameter	θ	30.9
capital diffusion volatility	σ	14.1%
mitigation technology parameter	β_1	842
jump arrival rate if State is B	λ_B	2
Targeted observables without mitigation (State G)		
(real) risk-free rate		0.8%
equity risk premium		6.6%
stock market return volatility		14.5%
consumption-investment ratio		2.84


A. $x(\pi)$
B. $i(\pi)$
C. $c(\pi)$
D. $q(\pi)$
planner
market
π
π
0.04
0.02
0
0.04
0.035
0.03
0.025
0.11
0.105
0.1
0.095
0.09
2.2
2.1
2
1.9
1.8
0
0.2
0.4
0.6
0.8
1

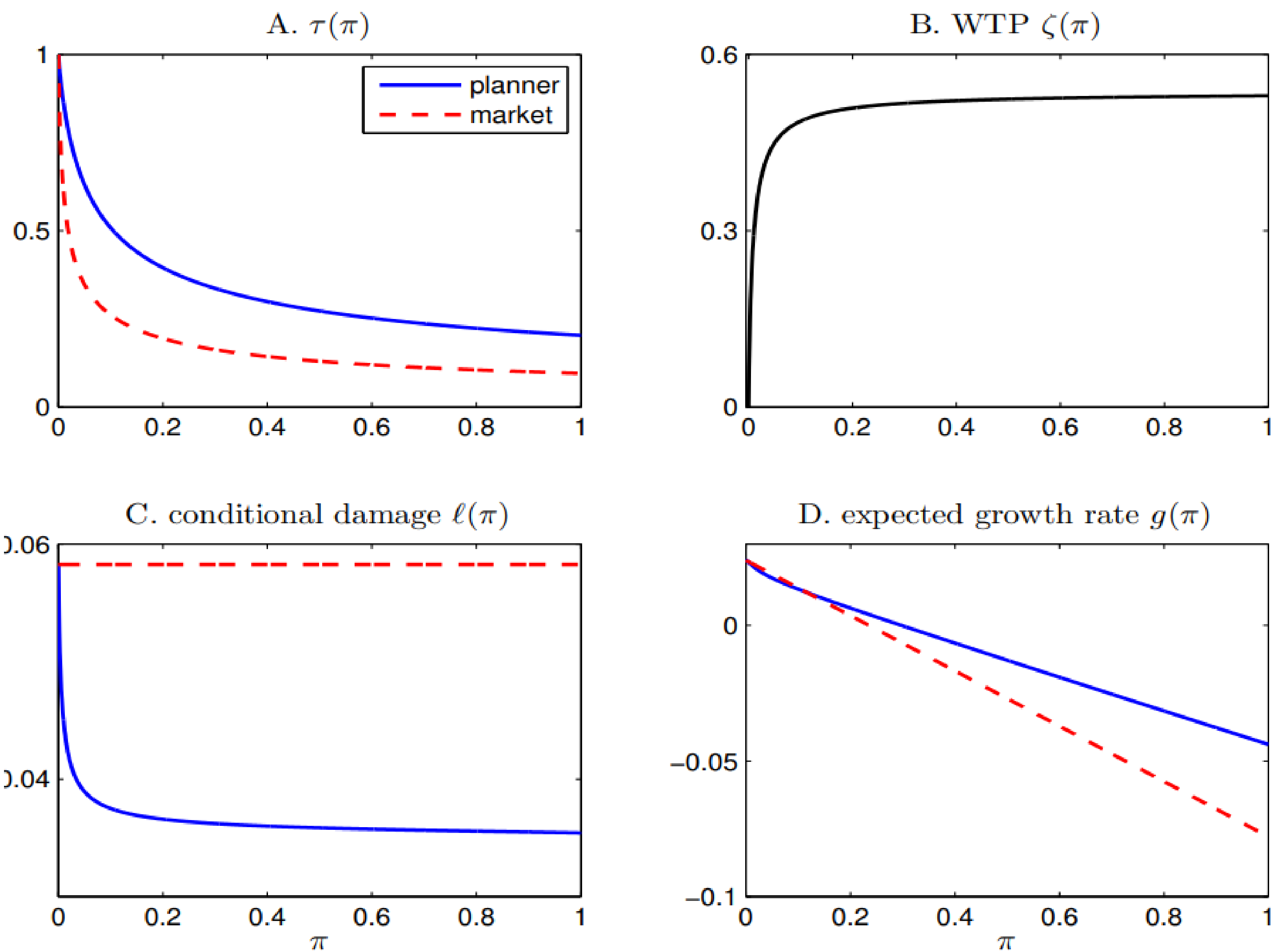

A. $\tau(\pi)$
planner
market
B. WTP $\zeta(\pi)$
C. conditional damage $\ell(\pi)$
D. expected growth rate $g(\pi)$
π
π

Table 2: The effect of mitigation efficacy β_1 and the disaster likelihood λ_B on WTP for the mitigation technology: $\zeta(\pi)$. We report the values for $\zeta(\pi)$ at two levels of belief: $\pi = 2\%$ and $\pi = 5\%$ ($\psi = 1$ in Panels A and C, and $\psi = 1.5$ in Panels B and D.)

	Panel A.	$\psi = 1$	Panel B.	$\psi = 1.5$
β_1	$\pi = 2\%$	$\pi = 5\%$	$\pi = 2\%$	$\pi = 5\%$
0	0	0	0	0
200	8.27%	9.66%	0.56%	0.83%
400	24.4%	29.0%	5.40%	8.28%
842	**37.1%**	**44.9%**	11.6%	18.0%
1600	43.7%	53.4%	16.5%	25.0%

	Panel C.	$\psi = 1$	Panel D.	$\psi = 1.5$
λ_B	$\pi = 2\%$	$\pi = 5\%$	$\pi = 2\%$	$\pi = 5\%$
0.25	0	0	0	0
1	3.50%	6.79%	1.58%	3.26%
2	**37.1%**	**44.9%**	11.6%	18.0%
4	86.3%	86.6%	39.6%	44.0%

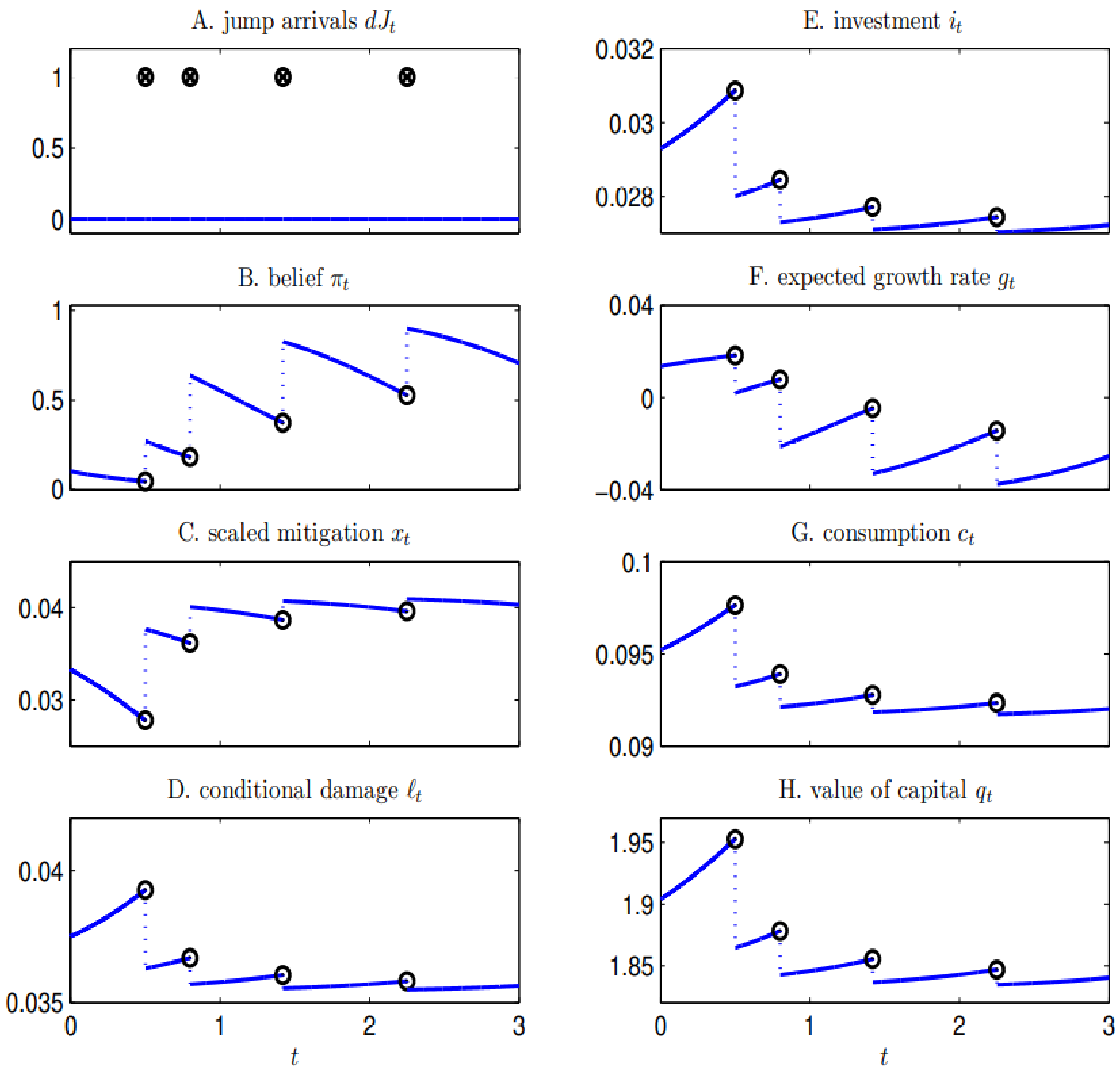


Figure 5: This figure simulates a jump path with four arrivals at $t = 0.5, 0.81, 1.42, 2.25$ (with $\pi_0 = 0.1$) and plots dynamics for belief updating, policies, and valuation.

Regulatory Risks and Sustainable Investing

- Direct effect： Increasing mitigation funded by taxes to curtail the damage of disaster risk causes further prolonged drops in consumption and investment, leading to lower expected growth rates and stock market valuations.

- Indirect effect: indirect effect of belief updating that disasters in the future are more likely and hence investment opportunities are worse.

Conclusion

- We provide the planner s solution to a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate potential future damages.
- Mitigation—by curtailing aggregate risk and insuring sustainable growth—is a public good in our model. The planner's solution can be implemented via a capital tax and mitigation subsidy scheme.
- Our model provides an integrated assessment of the cost and benefit of mitigation efforts such as public health spending or environmental protection via an aggregate risk management rationale.
- Our model also delivers a number of testable implications pertaining to damage functions, regulatory risks and sustainable investments. Future research avenues include an estimation of our model using damage and mitigation spending data.

Thank you